* 1. **Wave-Based Modelling**
     1. **Introduction**

Conventional audio rendering approaches are based on geometric or ray-based approximations as we have already discussed in previous chapters. These, though, conceptually simple, however, such methods ignore many audible features of room acoustics, particular are not valid at low audible frequencies. In other word, GA methods are not capable of rendering diffraction or shadowing effects around corners or obstructions. Therefore, Wave-based methods address the shortcomings of ray-based methods and yield a complete description of the acoustic field within a closed space. The available used wave-based methods are, generally, categorized into three main parts based on the techniques used. These are; **1)** finite element (FE), **2)** boundary element (BE), and **3)** finite-difference time-domain (FDTD). Both FE and FDTD methods discretized the closed spaces into volumetric elements, whereas, BE method discretized the boundaries of the closed spaces which are under investigations. The computational cost of these methods depends directly on the number of elements in the discretized mesh. Smaller is the element size the more elements are processed and in turn the wider is the covered bandwidth. Typically, above mentioned methods are employed for modelling of low frequency behaviour of rooms as these are more accurate at low frequencies. On the other hand geometric approach is used for higher frequencies, especially above the Schroder frequency (see Chapter **ISM/RT**). Both finite element and boundary element approached, generally, operate in the frequency domain, therefore, are best suitable for the modelling of static room acoustical scenes, such that the source and receiver are fixed in its positions and are not allowed to move during the computations. Out of these methods, the FDTD technique is the most suitable for real-time auralization. With a precomputation step even the BEM results can be used for auralization of static scenes. On the other hand, in FDTD the simulation progresses iteratively in time steps. In traditional FDTD methods two variables, which are the sound pressure and the volume velocity are desirable and its values are updated alternately [**2**], however, it is promising to use only one variable as well. The FDTD schemes are divided into two parts, namely the explicit schemes and the implicit schemes. The explicit schemes are straightforward to implement as the values of the next time step are computed solely based on the previous time steps. The implicit FDTD schemes are more complicated as the new value of a node depends on the new values of neighbouring nodes thus requiring simultaneous solving of all nodes. In all available FDTD schemes the numeric dispersion is an inherent problem which affects the bandwidth and is considered as valid. In addition to the dispersion another challenge in FDTD modelling is frequency dependent boundary conditions, however, there is a recent well-described solution termed as digital impedance filters. In the next sections we will discuss the Wave-based techniques and their working principles.

* + 1. **Boundary Element Method**

The boundary element method (BEM) is explicitly related to Green’s function with denotes source position and are a set of field points. The radiation problem, however, is rearranged into the Helmholtz-Kirchhoff integral equation and discretized. The Helmholtz-Kirchhoff integral is the source free Helmholtz-Huygens integral in Equation **3.1**.

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|  | **(3.1)** |

with Green’s functions in 3D free space given in Equation 3.2, which fulfil far-field radiation (known as Sommerfeld) condition of disappearing sound pressure as .

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|  | **(3.2)** |

The kernel of this integral contains monopole as well as dipole sources. The main application for BEM is radiation or equivalent radiation problems such as scattering [**1**]. Radiation problems are characterized by boundary conditions (which are local impedances or admittances) including a vibrational velocity as a driving source. This integral is formulated in discretized form on a surface mesh and solved numerically with matrix algebra. The crucial point of the BEM formulation is the numerical non-uniqueness. It important to note that in contrast to FEM matrices, the BEM matrices are complete. Burton and Miller in 1971 presented an approach where these problems are discussed in details. Another strategy for avoiding numerical problems is the so-called CHIEF point method (i.e. combined integral equation formulation. The complexity of BEM depend on the following factor given as;

1. A simulation which must be calculated up to a frequency of required a mesh element size of at most . The resulting model size of a surface is given is Equation 3.2.

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|  | **(3.3)** |

1. The BEM matrix contains entries, hence, the BEM solvers for PCs are capable of inverting a matrix of nodes in seconds. This result holds for one frequency only. In a problem of required frequency resolution of , calculation time multiplies by , which yields some hours for numerically generating a complete set of transfer functions for all field points.

An example HRTF computation using BEM simulation is shown in Figure **3.1**, taken from [**1**]. Advanced techniques such as fast multipole BEM are developments that allow separating meshes into regions of high discretization and others with the effect of transfer propagation. The complex linking between mesh elements is thus rearranged in a hierarchical way.

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| **Figure 3.1:** BEM-calculated HRTF of a customized dummy head: (left) Frontal incidence, (right) 45° in horizontal plane for the ipsilateral ear. |

* + 1. **Finite Element Method**

As mentioned in Section 3.11.1, in FE methods the finite elements are created by discretization of a sound field volume into small volume elements. In these elements the energy formulation of the harmonic field equations is used. This is generally known as Hamilton’s principle of minimum energy [**1**]. Any disturbance of the system equilibrium leads back to a stable and minimum energy state. Due to its general energetic formulation this principle is used for mechanical problems of static load and deformation (also for crash test simulation), for fluid dynamics, heat conduction, electromagnetic or acoustic field problems, and it is also the basis for the finite element method (FEM). As explained in [**1**], the acoustics space for any acoustic problem must be discretized into suitable volume elements. For each element the relation between forces and displacements is introduced by using the variational approach. Thus the variational approach is used to identify the field quantities for minimum energy element by element. The total energy is thus the sum of all the energies of the elements.

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| **Figure 3.2:** Example of an ear canal impedance 20log(|*Z*|/*Z*0) (for the CAD model shown, ear canal entrance and reference plane on the left) calculated using FEM |

In every element, the so-called ‘shape functions’ are defined to represent the sound pressures within the elements. At nodes between the elements the shape functions must fit continuously. All elements’ entries are combined into a so-called ‘stiffness’ matrix , a mass matrix and a damping matrix . Furthermore, source contributions and boundary conditions are formulated and integrated into a matrix equation including , and which is to be solved to obtain the sound pressures in the field space from the shape functions read at their nodes.

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| **Figure 3.3:** Modulus and phase of results from FEM calculations and measurements in a recording studio. Example from [**3**] |

In FEM solvers the direct solution to determine the eigenvalues of the matrix can be used by using the matrix equation without further subspace conditions. In the indirect method, the problem is projected on a modal basis into an equivalent eigenvalue problem of orthogonal modes. The indirect method has the great advantage that sources and boundary conditions can be studied in a second step. The numerical complexity is then given by the size of the modal basis and not by the FE mesh size. State of the art in FE is used for sound pressure calculation in small or midsize rooms for frequencies up to few kHz. Using PC software, typical mesh sizes are in a range of 100,000 nodes, and typical calculation times are of the order of magnitude of 5 minutes per frequency [**1**].

* + 1. **Time domain models**

Wave propagation can be simulated in the time domain as well. Pulse propagation can now be studied in a mesh from node to node which is known as finite difference model or on a large scale assuming special wave types (i.e. ray tracing). The results are wave fronts propagating in time. Reflections, diffraction and other propagation effects can be calculated form this and at chosen field points, impulse responses can be synthesized.

* + - 1. **Waveguides**

The realization of wave propagation in one dimension (1D) is perfectly represented by waveguides. These waveguides introduce propagation delay and attenuation because of divergence and damping. An omnidirectional spherical wave can be described by waveguides. Points of reflections due to interfaces between different impedances are easy to implement by connecting waveguides which are the delay lines with transfer functions of reflection and transmission factors. It is obvious that waveguides contain back and forth travelling waves and hence are bidirectional. In 2-D or 3-D wave propagation the waveguide model is mapped to a corresponding CAD model such as a room. The delay lines then are geometrically fixed at some point or patches on the walls, otherwise, the combinations of geometric paths would increase exponentially. Waveguides are well described and studied in application to physical modelling of musical instruments and vocal tracts [**4-6**]. In these cases, the transmission system is separated into adjacent tubes of varying cross section. Frequency dependent losses are included adding digital low pass filters into the delay loop.

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| **Figure 3.4:** (**a**) Mapping room geometry on a set of coupled delay lines and (**b**) Room model with waveguide network |

FIR and IIR filter networks are used, partly involving sophisticated phase models and subsample shifts for better adjusting the actual geometric relations between nodes. The geometric conditions and corresponding wave divergence must be specifically included, except for 1-D application such as for wind instruments and vocal tracts [**1**].